Homework 5 STAT 351

Chapter 5

1. **An instructor has given a short test consisting of two parts. For a randomly selected student, let X = the number of points earned on the first part and Y = the number of points earned on the second part. Suppose that the joint pmf of X and Y is given in the accompanying table.**

Value of $p(x,y) = P(X=x, Y=y)$

		y				
$p(x, y)$		0	5	10	15	
	0	.02	.06	.02	.10	Σ 0.20
x	5	.04	.15	.20	.10	Σ 0.49
	10	.01	.15	.14	.01	Σ 0.31
		Σ 0.07	Σ 0.36	Σ 0.36	Σ 0.21	

Prob. mass fn.

(a) Obtain marginal pmf of X.

$p(0) = 0.07$

$p(5) = 0.36$

$p(10) = 0.36$

$p(15) = 0.21$

(b) Obtain marginal pmf of Y .

$p(0) = 0.2$

$p(5) = 0.49$

$p(10) = 0.31$

(c) What is E(X), E(Y)and E(XY)?

Expected Value

$E(X) = 0(0.7) + 5(0.36) + 10(0.36) + 15(0.21) = \boxed{8.55}$

$E(Y) = 0(0.2) + 5(0.49) + 10(0.31) = \boxed{5.55}$

$E(XY) = (5)\cdot(5)(0.15) + (5)\cdot(10)\cdot(0.15+0.2)$

(d) Given V (X) = 19.15 and V (Y) = 12, 45, calculate Cov(X, Y) and Corr(X, Y). What can you say about the relationship between the points earend for first part (X) and points earned for the second part (Y)?

(e) If the recorded scores were M = (maximum of X and Y), obtain pmf of M.

(f) Is X and Y independent? Support your answer by numerical evidence.

2. Let r.v. X and Y have joint pdf

$$f(x,y) = \begin{cases} \frac{1}{54}(x+y) & \text{for } 0 \le x \le 3 \text{ and } 1 \le y \le 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Obtain marginal pdf of X.

$$= \frac{4}{54}x + \frac{12}{54}$$

$$f(x) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_{1}^{5} \frac{1}{54}(x+y)\,dy \rightarrow \int_{1}^{5} \frac{1}{54}x + \frac{1}{54}y\,dy \rightarrow$$

$$\boxed{f(x) = \frac{4}{54}x + \frac{12}{54}}$$

(b) Calculate E(X).

$$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{0}^{3} x\left(\frac{4}{54}x + \frac{12}{54}\right)dx = \left[\frac{4}{54}\left(\frac{x^3}{3}\right) + \frac{12}{54}\left(\frac{x^2}{2}\right)\right]\Bigg|_{0}^{3} = 1.67$$

(c) Calculate P

(d) Calculate E(XY)